

Volume

MODULE



13



ESSENTIAL QUESTION

How can you use volume to solve real-world problems?



LESSON 13.1

Volume of Cylinders

FL 8.G.3.9

LESSON 13.2

Volume of Cones

FL 8.G.3.9

LESSON 13.3

Volume of Spheres

FL 8.G.3.9



Real-World Video

Many foods are in the shape of cylinders, cones, and spheres. To find out how much of the food you are eating, you can use formulas for volume.

my.hrw.com

© Houghton Mifflin Harcourt Publishing Company

GO
DIGITAL
my.hrw.com



my.hrw.com

Go digital with your write-in student edition, accessible on any device.



Math On the Spot

Scan with your smart phone to jump directly to the online edition, video tutor, and more.



Animated Math

Interactively explore key concepts to see how math works.



Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.



Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Exponents

EXAMPLE $6^3 = 6 \times 6 \times 6$
 $= 36 \times 6$
 $= 216$

Multiply the base (6) by itself the number of times indicated by the exponent (3).

Find the product of the first two terms.

Find the product of all the terms.

Evaluate each exponential expression.

1. 11^2 _____ 2. 2^5 _____ 3. $\left(\frac{1}{5}\right)^3$ _____ 4. $(0.3)^2$ _____
5. 2.1^3 _____ 6. 0.1^3 _____ 7. $\left(\frac{9.6}{3}\right)^2$ _____ 8. 100^3 _____

Round Decimals

EXAMPLE Round 43.2685 to the underlined place.
 $43.2685 \rightarrow 43.27$

The digit to be rounded: 6
The digit to its right is 8.
8 is 5 or greater, so round up.
The rounded number is 43.27.

Round to the underlined place.

9. 2.374 _____ 10. 126.399 _____ 11. 13.9577 _____ 12. 42.690 _____
13. 134.95 _____ 14. 2.0486 _____ 15. 63.6352 _____ 16. 98.9499 _____

Simplify Numerical Expressions

EXAMPLE $\frac{1}{3}(3.14)(4)^2(3) = \frac{1}{3}(3.14)(16)(3)$ Simplify the exponent.
 $= 50.24$ Multiply from left to right.

Simplify each expression.

17. $3.14(5)^2(10)$ _____ 18. $\frac{1}{3}(3.14)(3)^2(5)$ _____ 19. $\frac{4}{3}(3.14)(3)^3$ _____
20. $\frac{4}{3}(3.14)(6)^3$ _____ 21. $3.14(4)^2(9)$ _____ 22. $\frac{1}{3}(3.14)(9)^2\left(\frac{2}{3}\right)$ _____

Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the empty columns in the chart. You may use words more than once.

Shape	Distance Around	Attributes	Associated Review Words
circle		r, d	
square		90° corner, sides	
rectangle		90° corner, sides	

Vocabulary

Review Words

- area (*área*)
- base (*base, en numeración*)
- ✓ circumference (*circunferencia*)
- ✓ diameter (*diámetro*)
- height (*altura*)
- ✓ length (*longitud*)
- ✓ perimeter (*perímetro*)
- ✓ radius (*radio*)
- ✓ right angle (*ángulo recto*)
- ✓ width (*ancho*)

Preview Words

- cone (*cono*)
- cylinder (*cilindro*)
- sphere (*esfera*)

Understand Vocabulary

Complete the sentences using the preview words.

1. A three-dimensional figure that has one vertex and one circular base is a _____.
2. A three-dimensional figure with all points the same distance from the center is a _____.
3. A three-dimensional figure that has two congruent circular bases is a _____.

Active Reading

Three-Panel Flip Chart Before beginning the module, create a three-panel flip chart to help you organize what you learn. Label each flap with one of the lesson titles from this module. As you study each lesson, write important ideas like vocabulary, properties, and formulas under the appropriate flap.





Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

FL 8.G.3.9

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Key Vocabulary**volume** (*volumen*)

The number of cubic units needed to fill a given space.

cylinder (*cilindro*)

A three-dimensional figure with two parallel, congruent circular bases connected by a curved lateral surface.

What It Means to You

You will learn the formula for the volume of a cylinder.

UNPACKING EXAMPLE 8.G.3.9

The Asano Taiko Company of Japan built the world's largest drum in 2000. The drum's diameter is 4.8 meters, and its height is 4.95 meters. Estimate the volume of the drum.

$$d = 4.8 \approx 5$$

$$h = 4.95 \approx 5$$

$$r = \frac{d}{2} \approx \frac{5}{2} = 2.5$$

$$V = (\pi r^2)h$$

$$\approx (3) (2.5)^2 \cdot 5$$

$$= (3) (6.25) (5)$$

$$= 18.75 \cdot 5$$

$$= 93.75 \approx 94$$

Volume of a cylinder

Use 3 for π .

The volume of the drum is approximately 94 m^3 .

FL 8.G.3.9

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Key Vocabulary**cone** (*cono*)

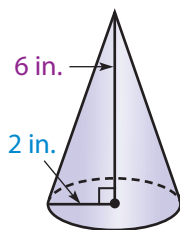
A three-dimensional figure with one vertex and one circular base.

sphere (*esfera*)

A three-dimensional figure with all points the same distance from the center.

What It Means to You

You will learn formulas for the volume of a cone and a sphere.

UNPACKING EXAMPLE 8.G.3.9

Find the volume of the cone. Use 3.14 for π .

$$B = \pi(2^2) = 4\pi \text{ in}^2$$

$$V = \frac{1}{3} \cdot 4\pi \cdot 6$$

$$V = 8\pi$$

$$\approx 25.1 \text{ in}^3$$

$$V = \frac{1}{3} Bh$$

Use 3.14 for π .

The volume of the cone is approximately 25.1 in^3 .

The volume of a sphere with the same radius is

$$V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}(3)(2)^3 = 32 \text{ in}^3.$$



Visit my.hrw.com to see all **Florida Math Standards** unpacked.

13.1 Volume of Cylinders



Know the formulas for the volumes of... cylinders... and use them to solve real-world and mathematical problems.



ESSENTIAL QUESTION

How do you find the volume of a cylinder?

EXPLORE ACTIVITY



Modeling the Volume of a Cylinder

A **cylinder** is a three-dimensional figure that has two congruent circular bases that lie in parallel planes. The volume of any three-dimensional figure is the number of cubic units needed to fill the space taken up by the solid figure.

One cube represents one cubic unit of volume. You can develop the formula for the volume of a cylinder using an empty soup can or other cylindrical container. First, remove one of the bases.

- A** Arrange centimeter cubes in a single layer at the bottom of the cylinder. Fit as many cubes into the layer as possible. How many cubes are in this layer?

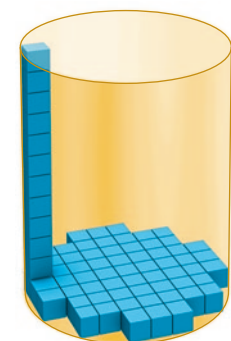
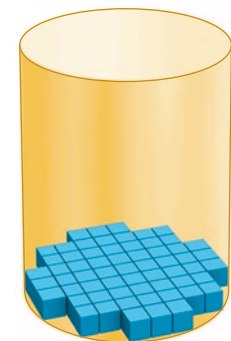
- B** To find how many layers of cubes fit in the cylinder, make a stack of cubes along the inside of the cylinder. How many layers fit in the cylinder?

- C** How can you use what you know to find the approximate number of cubes that would fit in the cylinder?

Reflect

- 1. Make a Conjecture** Suppose you know the area of the base of a cylinder and the height of the cylinder. How can you find the cylinder's volume?

- 2.** Let the area of the base of a cylinder be B and the height of the cylinder be h . Write a formula for the cylinder's volume V . _____





Math On the Spot

my.hrw.com

Finding the Volume of a Cylinder Using a Formula

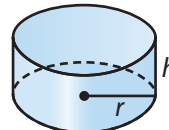
Finding volumes of cylinders is similar to finding volumes of prisms. You find the volume V of both a prism and a cylinder by multiplying the height h by the area of the base B , so $V = Bh$.

The base of a cylinder is a circle, so for a cylinder, $B = \pi r^2$.

Volume of a Cylinder

The volume V of a cylinder with radius r is the area of the base B times the height h .

$$V = Bh \text{ or } V = \pi r^2 h$$



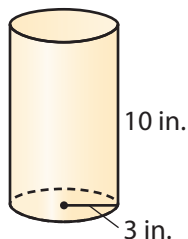
EXAMPLE 1



FL 8.G.3.9

Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π .

A



$$V = \pi r^2 h$$

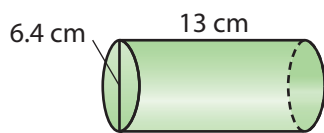
$$\approx 3.14 \cdot 3^2 \cdot 10 \quad \text{Substitute.}$$

$$\approx 3.14 \cdot 9 \cdot 10 \quad \text{Simplify.}$$

$$\approx 282.6 \quad \text{Multiply.}$$

The volume is about 282.6 in^3 .

B



Since the diameter is 6.4 cm, the radius is 3.2 cm.

$$V = \pi r^2 h$$

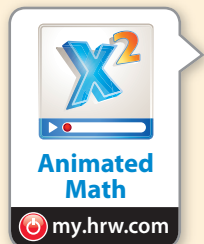
$$\approx 3.14 \cdot 3.2^2 \cdot 13 \quad \text{Substitute.}$$

$$\approx 3.14 \cdot 10.24 \cdot 13 \quad \text{Simplify.}$$

$$\approx 418 \quad \text{Multiply.}$$

The volume is about 418 cm^3 .

Recall that the diameter of a circle is twice the radius, so $2r = d$ and $r = \frac{d}{2}$.



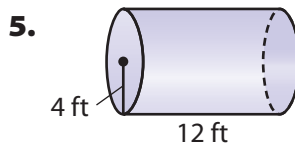
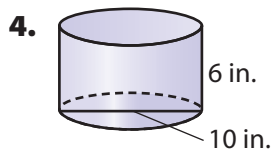
My Notes

Reflect

3. **What If?** If you want a formula for the volume of a cylinder that involves the diameter d instead of the radius r , how can you rewrite it?

YOUR TURN

Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π .



Finding the Volume of a Cylinder in a Real-World Context

The Longhorn Band at the University of Texas at Austin has one of the world's largest bass drums, known as Big Bertha.

EXAMPLE 2



FL 8.G.3.9

Big Bertha has a diameter of 8 feet and is 4.5 feet deep. Find the volume of the drum to the nearest tenth. Use 3.14 for π .

STEP 1 Find the radius of the drum.

$$r = \frac{d}{2} = \frac{8}{2} = 4 \text{ ft}$$

STEP 2 Find the volume of the drum.

$$V = \pi r^2 h$$

$$\approx 3.14 \cdot 4^2 \cdot 4.5$$

Substitute.

$$\approx 3.14 \cdot 16 \cdot 4.5$$

Simplify the exponent.

$$\approx 226.08$$

Multiply.

The volume of the drum is about 226.1 ft^3 .



YOUR TURN

6. A drum company advertises a snare drum that is 4 inches high and 12 inches in diameter. Find the volume of the drum to the nearest tenth. Use 3.14 for π .



Personal
Math Trainer

Online Assessment
and Intervention

my.hrw.com



Math On the Spot

my.hrw.com



Personal
Math Trainer

Online Assessment
and Intervention

my.hrw.com

Guided Practice

1. **Vocabulary** Describe the bases of a cylinder. (Explore Activity)

2. Figure 1 shows a view from above of inch cubes on the bottom of a cylinder. Figure 2 shows the highest stack of cubes that will fit inside the cylinder. Estimate the volume of the cylinder. Explain your reasoning. (Explore Activity)

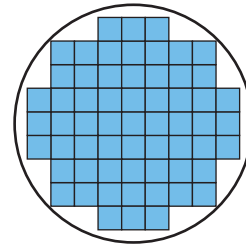


Figure 1



Figure 2

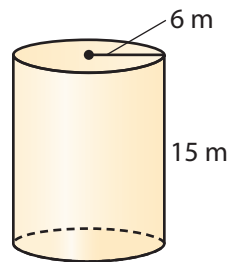
3. Find the volume of the cylinder to the nearest tenth. Use 3.14 for π . (Example 1)

$$V = \pi r^2 h$$

$$V = \pi \cdot \boxed{}^2 \cdot \boxed{}$$

$$\approx 3.14 \cdot \boxed{} \cdot \boxed{}$$

$$\approx \boxed{}$$



The volume of the cylinder is approximately _____ m^3 .

4. A Japanese odaiko is a very large drum that is made by hollowing out a section of a tree trunk. A museum in Takayama City has three odaikos of similar size carved from a single tree trunk. The largest measures about 2.7 meters in both diameter and length, and weighs about 4.5 metric tons. Using the volume formula for a cylinder, approximate the volume of the drum to the nearest tenth. (Example 2)

The radius of the drum is about _____ m.

The volume of the drum is about _____ m^3 .




ESSENTIAL QUESTION CHECK-IN

5. How do you find the volume of a cylinder? Describe which measurements of a cylinder you need to know.


13.1 Independent Practice

 **FL** 8.G.3.9

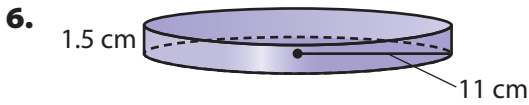


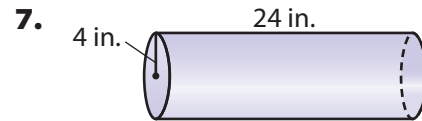
Personal Math Trainer

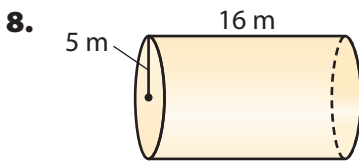
Online Assessment and Intervention

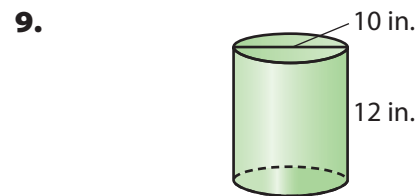
 my.hrw.com

Find the volume of each figure. Round your answers to the nearest tenth if necessary. Use 3.14 for π .









10. A cylinder has a radius of 4 centimeters and a height of 40 centimeters.

11. A cylinder has a radius of 8 meters and a height of 4 meters.

Round your answer to the nearest tenth, if necessary. Use 3.14 for π .

12. The cylindrical Giant Ocean Tank at the New England Aquarium in Boston is 24 feet deep and has a radius of 18.8 feet. Find the volume of the tank.

13. A standard-size bass drum has a diameter of 22 inches and is 18 inches deep. Find the volume of this drum.

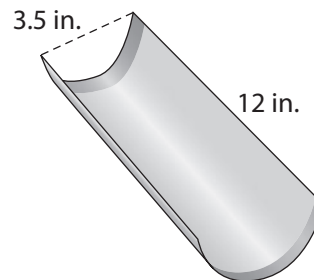
14. Grain is stored in cylindrical structures called silos. Find the volume of a silo with a diameter of 11.1 feet and a height of 20 feet.

15. The Frank Erwin Center, or “The Drum,” at the University of Texas in Austin can be approximated by a cylinder that is 120 meters in diameter and 30 meters in height. Find its volume.



16. A barrel of crude oil contains about 5.61 cubic feet of oil. How many barrels of oil are contained in 1 mile (5280 feet) of a pipeline that has an inside diameter of 6 inches and is completely filled with oil? How much is "1 mile" of oil in this pipeline worth at a price of \$100 per barrel?

17. A pan for baking French bread is shaped like half a cylinder. It is 12 inches long and 3.5 inches in diameter. What is the volume of uncooked dough that would fill this pan?



FOCUS ON HIGHER ORDER THINKING

18. **Explain the Error** A student said the volume of a cylinder with a 3-inch diameter is two times the volume of a cylinder with the same height and a 1.5-inch radius. What is the error?

19. **Communicate Mathematical Ideas** Explain how you can find the height of a cylinder if you know the diameter and the volume. Include an example with your explanation.

20. **Analyze Relationships** Cylinder A has a radius of 6 centimeters. Cylinder B has the same height and a radius half as long as cylinder A. What fraction of the volume of cylinder A is the volume of cylinder B? Explain.

Work Area

LESSON

13.2 Volume of Cones

 **FL** 8.G.3.9

Know the formulas for the volumes of cones... and use them to solve real-world and mathematical problems.



ESSENTIAL QUESTION

How do you find the volume of a cone?

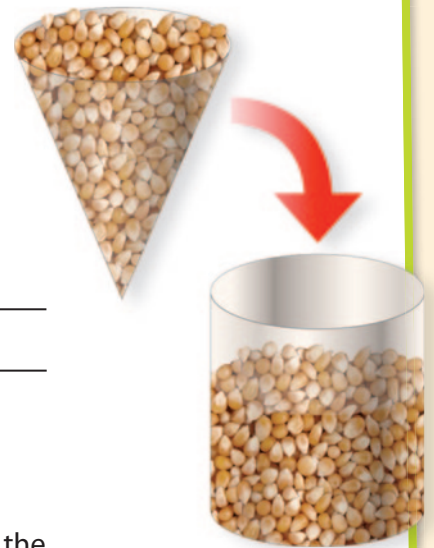
EXPLORE ACTIVITY

 **FL** 8.G.3.9

Modeling the Volume of a Cone

A **cone** is a three-dimensional figure that has one vertex and one circular base.

To explore the volume of a cone, Sandi does an experiment with a cone and a cylinder that have congruent bases and heights. She fills the cone with popcorn kernels and then pours the kernels into the cylinder. She repeats this until the cylinder is full.



Sandi finds that it takes **3 cones** to fill the volume of the cylinder.

STEP 1 What is the formula for the volume V of a cylinder with base area B and height h ? _____

STEP 2 What is the area of the base of the cone? _____

STEP 3 Sandi found that, when the bases and height are the same, _____ times $V_{\text{cone}} = V_{\text{cylinder}}$.

STEP 4 How does the volume of the cone compare to the volume of the cylinder?

Volume of the cone: $V_{\text{cone}} = \frac{\square}{\square} \cdot V_{\text{cylinder}}$

Reflect

- Use the conclusion from this experiment to write a formula for the volume of a cone in terms of the height and the radius. Explain.

- How do you think the formula for the volume of a cone is similar to the formula for the volume of a pyramid?



Math On the Spot

my.hrw.com

Finding the Volume of a Cone Using a Formula

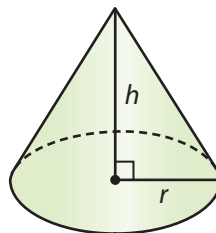
The formulas for the volume of a prism and the volume of a cylinder are the same: multiply the height h by the area of the base B , so $V = Bh$.

In the **Explore Activity**, you saw that the volume of a cone is one third the volume of a cylinder with the same base and height.

Volume of a Cone

The volume V of a cone with radius r is one third the area of the base B times the height h .

$$V = \frac{1}{3} Bh \text{ or } V = \frac{1}{3} \pi r^2 h$$



My Notes

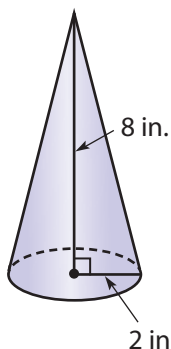
EXAMPLE 1



FL 8.G.3.9

Find the volume of each cone. Round your answers to the nearest tenth. Use 3.14 for π .

A



$$V = \frac{1}{3} \pi r^2 h$$

$$\approx \frac{1}{3} \cdot 3.14 \cdot 2^2 \cdot 8$$

Substitute.

$$\approx \frac{1}{3} \cdot 3.14 \cdot 4 \cdot 8$$

Simplify.

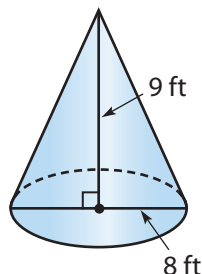
$$\approx 33.5$$

Multiply.

The volume is about 33.5 in^3 .

B

Since the diameter is 8 ft, the radius is 4 ft.



$$V = \frac{1}{3} \pi r^2 h$$

$$\approx \frac{1}{3} \cdot 3.14 \cdot 4^2 \cdot 9$$

Substitute.

$$\approx \frac{1}{3} \cdot 3.14 \cdot 16 \cdot 9$$

Simplify.

$$\approx 150.7$$

Multiply.

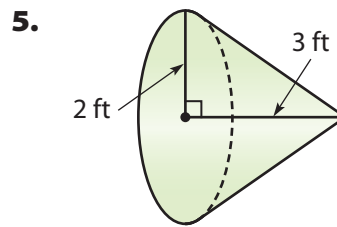
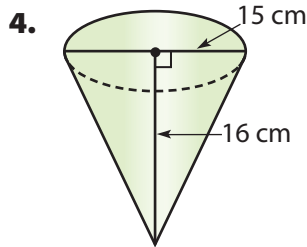
The volume is about 150.7 ft^3 .

Reflect

3. How can you rewrite the formula for the volume of a cone using the diameter d instead of the radius r ? _____

YOUR TURN

Find the volume of each cone. Round your answers to the nearest tenth.
Use 3.14 for π .



Finding the Volume of a Volcano

The mountain created by a volcano is often cone-shaped.

EXAMPLE 2



FL 8.G.3.9

For her geography project, Karen built a clay model of a volcano in the shape of a cone. Her model has a diameter of 12 inches and a height of 8 inches. Find the volume of clay in her model to the nearest tenth. Use 3.14 for π .

STEP 1 Find the radius.

$$r = \frac{12}{2} = 6 \text{ in.}$$

STEP 2 Find the volume of clay.

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ &\approx \frac{1}{3} \cdot 3.14 \cdot 6^2 \cdot 8 && \text{Substitute.} \\ &\approx \frac{1}{3} \cdot 3.14 \cdot 36 \cdot 8 && \text{Simplify.} \\ &\approx 301.44 && \text{Multiply.} \end{aligned}$$

○ The volume of the clay is about 301.4 in^3 .



Personal
Math Trainer

Online Assessment
and Intervention

my.hrw.com



Math On the Spot

my.hrw.com



YOUR TURN

6. The cone of the volcano Parícutin in Mexico had a height of 410 meters and a diameter of 424 meters. Approximate the volume of the cone.



Personal
Math Trainer

Online Assessment
and Intervention

my.hrw.com

Guided Practice

1. The area of the base of a cylinder is 45 square inches and its height is 10 inches. A cone has the same area for its base and the same height. What is the volume of the cone? [\(Explore Activity\)](#)

$$V_{\text{cylinder}} = Bh = \square \cdot \square = \square$$

$$V_{\text{cone}} = \frac{1}{3} V_{\text{cylinder}}$$

$$= \frac{1}{3} \square$$

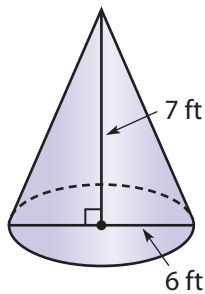
$$= \square$$

The volume of the cone is _____ in³.

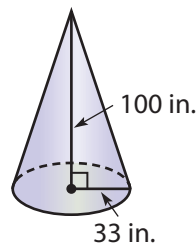
2. A cone and a cylinder have congruent height and bases. The volume of the cone is 18 m³. What is the volume of the cylinder? Explain. [\(Explore Activity\)](#)

Find the volume of each cone. Round your answer to the nearest tenth if necessary. Use 3.14 for π . [\(Example 1\)](#)

3.



4.



5. Gretchen made a paper cone to hold a gift for a friend. The paper cone was 15 inches high and had a radius of 3 inches. Find the volume of the paper cone to the nearest tenth. Use 3.14 for π . [\(Example 2\)](#)
6. A cone-shaped building is commonly used to store sand. What would be the volume of a cone-shaped building with a diameter of 50 meters and a height of 20 meters? Round your answer to the nearest tenth. Use 3.14 for π . [\(Example 2\)](#)




ESSENTIAL QUESTION CHECK-IN

7. How do you find the volume of a cone?

13.2 Independent Practice



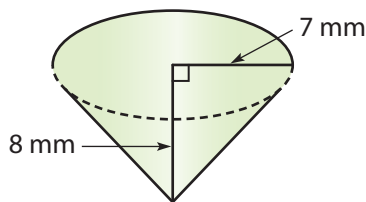
FL 8.G.3.9



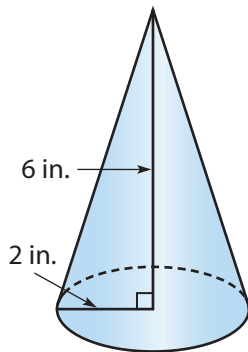
Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Find the volume of each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π .

8.



9.



10. A cone has a diameter of 6 centimeters and a height of 11.5 centimeters.

11. A cone has a radius of 3 meters and a height of 10 meters.

Round your answers to the nearest tenth if necessary. Use 3.14 for π .

12. Antonio is making mini waffle cones. Each waffle cone is 3 inches high and has a radius of $\frac{3}{4}$ inch. What is the volume of a waffle cone?

13. A snack bar sells popcorn in cone-shaped containers. One container has a diameter of 8 inches and a height of 10 inches. How many cubic inches of popcorn does the container hold?

14. A volcanic cone has a diameter of 300 meters and a height of 150 meters. What is the volume of the cone?

15. **Multistep** Orange traffic cones come in a variety of sizes. Approximate the volume, in cubic inches, of a traffic cone that has a height of 2 feet and a diameter of 10 inches. Use 3.14 for π .

Find the missing measure for each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π .

16. radius = _____

height = 6 in.

volume = 100.48 in³

17. diameter = 6 cm

height = _____

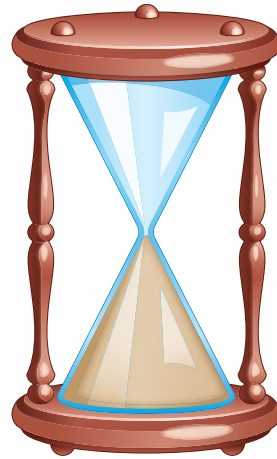
volume = 56.52 cm³

18. The diameter of a cone-shaped container is 4 inches, and its height is 6 inches. How much greater is the volume of a cylinder-shaped container with the same diameter and height? Round your answer to the nearest hundredth. Use 3.14 for π .



FOCUS ON HIGHER ORDER THINKING

Work Area



19. Alex wants to know the volume of sand in an hourglass. When all the sand is in the bottom, he stands a ruler up beside the hourglass and estimates the height of the cone of sand.

a. What else does he need to measure to find the volume of sand?

b. Make a Conjecture If the volume of sand is increasing at a constant rate, is the height increasing at a constant rate? Explain.

20. Problem Solving The diameter of a cone is x cm, the height is 18 cm, and the volume is 301.44 cm^3 . What is x ? Use 3.14 for π .

21. Analyze Relationships A cone has a radius of 1 foot and a height of 2 feet. How many cones of liquid would it take to fill a cylinder with a diameter of 2 feet and a height of 2 feet? Explain.

22. Critique Reasoning Herb knows that the volume of a cone is one third that of a cylinder with the same base and height. He reasons that a cone with the same height as a given cylinder but 3 times the radius should therefore have the same volume as the cylinder, since $\frac{1}{3} \cdot 3 = 1$. Is Herb correct? Explain.

LESSON 13.3 Volume of Spheres

 **FL 8.G.3.9**

Know the formulas for the volumes of... spheres and use them to solve real-world and mathematical problems.



ESSENTIAL QUESTION

How do you find the volume of a sphere?

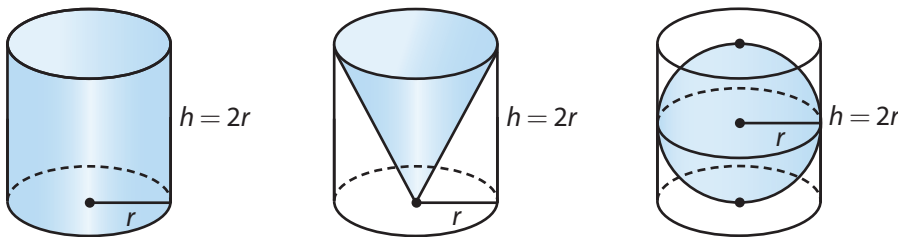
EXPLORE ACTIVITY

 **FL 8.G.3.9**

Modeling the Volume of a Sphere

A **sphere** is a three-dimensional figure with all points the same distance from the center. The **radius** of a sphere is the distance from the center to any point on the sphere.

You have seen that a cone fills $\frac{1}{3}$ of a cylinder of the same radius and height h . If you were to do a similar experiment with a sphere of the same radius, you would find that a sphere fills $\frac{2}{3}$ of the cylinder. The cylinder's height is equal to twice the radius of the sphere.



STEP 1 Write the formula $V = Bh$ for each shape. Use $B = \pi r^2$ and substitute the fractions you know for the cone and sphere.

Cylinder

$$V = \pi r^2 h$$

Cone

$$V = \frac{1}{3} \pi r^2 h$$

Sphere

$$V = \frac{2}{3} \pi r^2 h$$

STEP 2 Notice that a sphere always has a **height equal to twice the radius**. Substitute $2r$ for h .

$$V = \frac{2}{3} \pi r^2 (2r)$$

STEP 3 Simplify this formula for the volume of a sphere.

$$V = \boxed{} \pi r^3$$

Reflect

- Analyze Relationships** A cone has a radius of r and a height of $2r$. A sphere has a radius of r . Compare the volume of the sphere and cone.



Math On the Spot

my.hrw.com

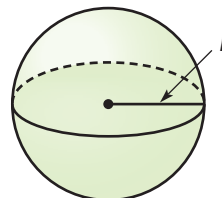
Finding the Volume of a Sphere Using a Formula

The Explore Activity illustrates a formula for the volume of a sphere with radius r .

Volume of a Sphere

The volume V of a sphere is $\frac{4}{3}\pi$ times the cube of the radius r .

$$V = \frac{4}{3}\pi r^3$$



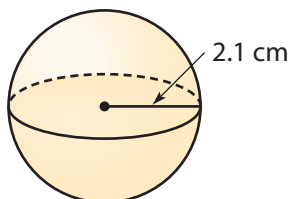
EXAMPLE 1



FL 8.G.3.9

Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π .

A



$$V = \frac{4}{3}\pi r^3$$

$$\approx \frac{4}{3} \cdot 3.14 \cdot 2.1^3$$

Substitute.

$$\approx \frac{4}{3} \cdot 3.14 \cdot 9.26$$

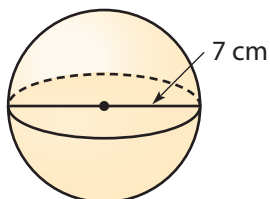
Simplify.

$$\approx 38.8$$

Multiply.

The volume is about 38.8 cm^3 .

B



Since the diameter is 7 cm, the radius is 3.5 cm.

$$V = \frac{4}{3}\pi r^3$$

$$\approx \frac{4}{3} \cdot 3.14 \cdot 3.5^3$$

Substitute.

$$\approx \frac{4}{3} \cdot 3.14 \cdot 42.9$$

Simplify.

$$\approx 179.6$$

Multiply.

The volume is about 179.6 cm^3 .

Math Talk

Mathematical Practices

If you know the diameter of a sphere, how would the formula for the volume of a sphere be written in terms of d ?

YOUR TURN

Find the volume of each sphere. Round your answers to the nearest tenth. Use 3.14 for π .

- A sphere has a radius of 10 centimeters. _____
- A sphere has a diameter of 3.4 meters. _____



Personal Math Trainer

Online Assessment and Intervention

my.hrw.com

Finding the Volume of a Sphere in a Real-World Context

Many sports, including golf and tennis, use a ball that is spherical in shape.



Math On the Spot

my.hrw.com

EXAMPLE 2



FL 8.G.3.9

Soccer balls come in several different sizes. One soccer ball has a diameter of 22 centimeters. What is the volume of this soccer ball? Round your answer to the nearest tenth. Use 3.14 for π .

STEP 1 Find the radius.

$$r = \frac{d}{2} = 11 \text{ cm}$$

STEP 2 Find the volume of the soccer ball.

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \\ &\approx \frac{4}{3} \cdot 3.14 \cdot 11^3 && \text{Substitute.} \\ &\approx \frac{4}{3} \cdot 3.14 \cdot 1331 && \text{Simplify.} \\ &\approx 5572.4533 && \text{Multiply.} \end{aligned}$$

The volume of the soccer ball is about 5572.5 cm^3 .



Reflect

4. What is the volume of the soccer ball in terms of π , to the nearest whole number multiple? Explain your answer.

5. **Analyze Relationships** The diameter of a basketball is about 1.1 times that of a soccer ball. The diameter of a tennis ball is about 0.3 times that of a soccer ball. How do the volumes of these balls compare to that of a soccer ball? Explain.

YOUR TURN

6. Val measures the diameter of a ball as 12 inches. How many cubic inches of air does this ball hold, to the nearest tenth? Use 3.14 for π .



Personal Math Trainer

Online Assessment and Intervention

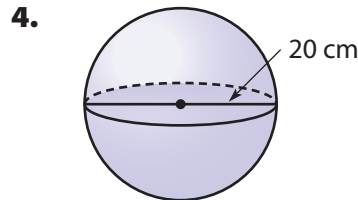
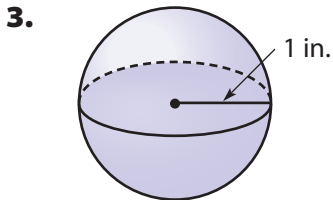
my.hrw.com

Guided Practice

1. **Vocabulary** A sphere is a three-dimensional figure with all points _____ from the center. (*Explore Activity*)

2. **Vocabulary** The _____ is the distance from the center of a sphere to a point on the sphere. (*Explore Activity*)

Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π . (*Example 1*)



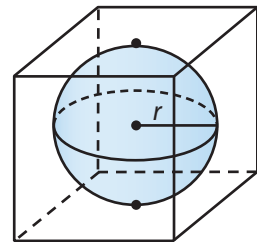
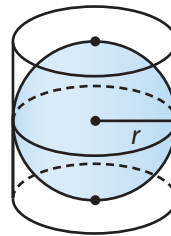
5. A sphere has a radius of 1.5 feet. _____

6. A sphere has a diameter of 2 yards. _____

7. A baseball has a diameter of 2.9 inches. Find the volume of the baseball. Round your answer to the nearest tenth if necessary. Use 3.14 for π . (*Example 2*) _____

8. A basketball has a radius of 4.7 inches. What is its volume to the nearest cubic inch. Use 3.14 for π . (*Example 2*) _____

9. A company is deciding whether to package a ball in a cubic box or a cylindrical box. In either case, the ball will touch the bottom, top, and sides. (*Explore Activity*)



a. What portion of the space inside the cylindrical box is empty? Explain.

b. Find an expression for the volume of the cubic box. _____


c. About what portion of the space inside the cubic box is empty? Explain.

ESSENTIAL QUESTION CHECK-IN

10. Explain the steps you use to find the volume of a sphere.

13.3 Independent Practice

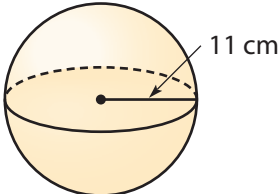
 **FL** 8.G.3.9

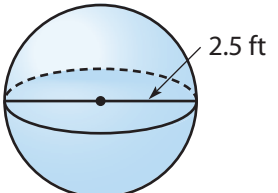


Personal Math Trainer
Online Assessment and Intervention
my.hrw.com

Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π .

- 11. radius of 3.1 meters _____
- 12. diameter of 18 inches _____
- 13. $r = 6$ in. _____
- 14. $d = 36$ m _____

15.  _____

16.  _____

The eggs of birds and other animals come in many different shapes and sizes. Eggs often have a shape that is nearly spherical. When this is true, you can use the formula for a sphere to find their volume.

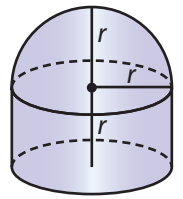
- 17. The green turtle lays eggs that are approximately spherical with an average diameter of 4.5 centimeters. Each turtle lays an average of 113 eggs at one time. Find the total volume of these eggs, to the nearest cubic centimeter.

- 18. Hummingbirds lay eggs that are nearly spherical and about 1 centimeter in diameter. Find the volume of an egg. Round your answer to the nearest tenth.

- 19. Fossilized spherical eggs of dinosaurs called titanosaurid sauropods were found in Patagonia. These eggs were 15 centimeters in diameter. Find the volume of an egg. Round your answer to the nearest tenth.

- 20. **Persevere in Problem Solving** An ostrich egg has about the same volume as a sphere with a diameter of 5 inches. If the eggshell is about $\frac{1}{12}$ inch thick, find the volume of just the shell, not including the interior of the egg. Round your answer to the nearest tenth.

- 21. **Multistep** Write the steps you would use to find a formula for the volume of the figure at right. Then write the formula.



22. **Critical Thinking** Explain what happens to the volume of a sphere if you double the radius.

23. **Multistep** A cylindrical can of tennis balls holds a stack of three balls so that they touch the can at the top, bottom, and sides. The radius of each ball is 1.25 inches. Find the volume inside the can that is not taken up by the three tennis balls.



FOCUS ON HIGHER ORDER THINKING

24. **Critique Reasoning** A sphere has a radius of 4 inches, and a cube-shaped box has an edge length of 7.5 inches. J.D. says the box has a greater volume, so the sphere will fit in the box. Is he correct? Explain.

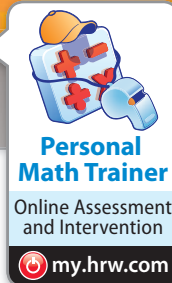
25. **Critical Thinking** Which would hold the most water: a bowl in the shape of a hemisphere with radius r , a cylindrical glass with radius r and height r , or a cone-shaped drinking cup with radius r and height r ? Explain.

26. **Analyze Relationships** Hari has models of a sphere, a cylinder, and a cone. The sphere's diameter and the cylinder's height are the same, $2r$. The cylinder has radius r . The cone has diameter $2r$ and height $2r$. Compare the volumes of the cone and the sphere to the volume of the cylinder.

27. A spherical helium balloon that is 8 feet in diameter can lift about 17 pounds. What does the diameter of a balloon need to be to lift a person who weighs 136 pounds? Explain.

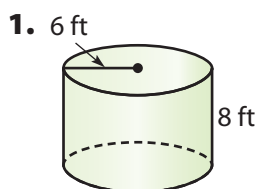
Work Area

Ready to Go On?



13.1 Volume of Cylinders

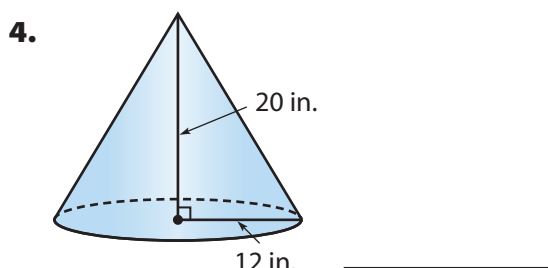
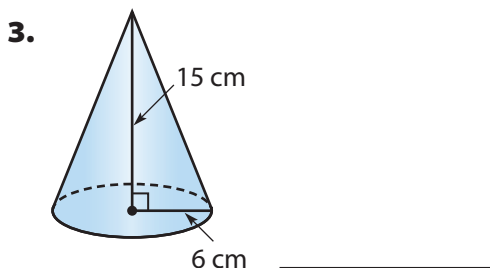
Find the volume of each cylinder. Round your answers to the nearest tenth if necessary. Use 3.14 for π .



2. A can of juice has a radius of 4 inches and a height of 7 inches. What is the volume of the can?

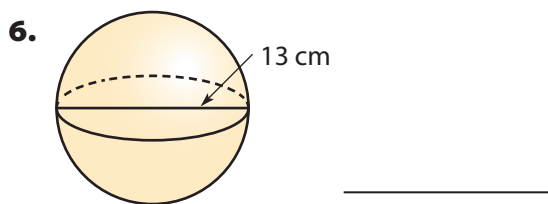
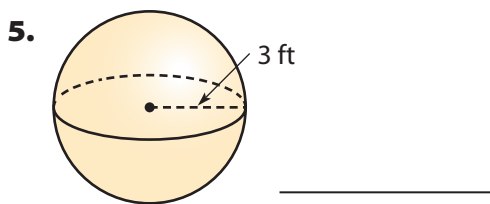
13.2 Volume of Cones

Find the volume of each cone. Round your answers to the nearest tenth if necessary. Use 3.14 for π .



13.3 Volume of Spheres

Find the volume of each sphere. Round your answers to the nearest tenth if necessary. Use 3.14 for π .



ESSENTIAL QUESTION

7. What measurements do you need to know to find the volume of a cylinder? a cone? a sphere?

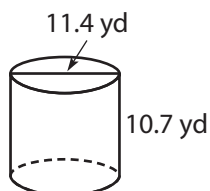


Selected Response

1. The bed of a pickup truck measures 4 feet by 8 feet. To the nearest inch, what is the length of the longest thin metal bar that will lie flat in the bed?

(A) 11 ft 3 in. (C) 8 ft 11 in.
(B) 10 ft 0 in. (D) 8 ft 9 in.

2. Using 3.14 for π , what is the volume of the cylinder below to the nearest tenth?



(A) 102 cubic yards
(B) 347.6 cubic yards
(C) 1,091.6 cubic yards
(D) 4,366.4 cubic yards

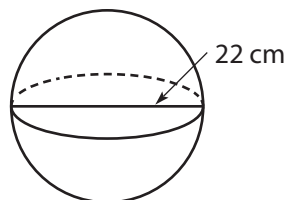
3. Rhett made mini waffle cones for a birthday party. Each waffle cone was 3.5 inches high and had a radius of 0.8 inches. What is the volume of each cone to the nearest hundredth?

(A) 1.70 cubic inches
(B) 2.24 cubic inches
(C) 2.34 cubic inches
(D) 8.79 cubic inches

4. What is the volume of a cone that has a height of 17 meters and a base with a radius of 6 meters? Use 3.14 for π and round to the nearest tenth.

(A) 204 cubic meters
(B) 640.6 cubic meters
(C) 2,562.2 cubic meters
(D) 10,249 cubic meters

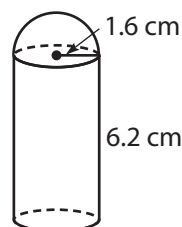
5. Using 3.14 for π , what is the volume of the sphere to the nearest tenth?



(A) 4180 cubic centimeters
(B) 5572.5 cubic centimeters
(C) 33,434.7 cubic centimeters
(D) 44,579.6 cubic centimeters

Mini-Task

6. A diagram of a deodorant container is shown. It is made up of a cylinder and half of a sphere.



Use 3.14 for π and round answers to the nearest tenth.

- a. What is the volume of the half sphere?

- b. What is the volume of the cylinder?

- c. What is the volume of the whole figure?

MODULE **11**

Angle Relationships in Parallel Lines and Triangles



ESSENTIAL QUESTION

How can you solve real-world problems that involve angle relationships in parallel lines and triangles?

EXAMPLE 1

Find each angle measure when $m\angle 6 = 81^\circ$.

A $m\angle 5 = 180^\circ - 81^\circ = 99^\circ$

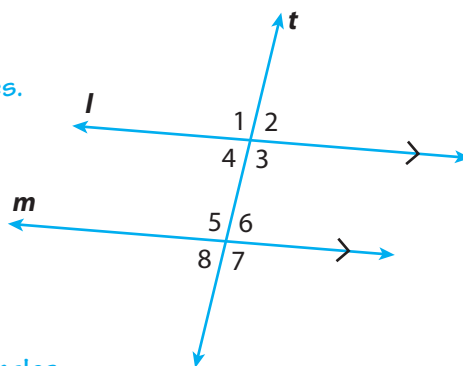
5 and 6 are supplementary angles.

B $m\angle 1 = 99^\circ$

1 and 5 are corresponding angles.

C $m\angle 3 = 180^\circ - 81^\circ = 99^\circ$

3 and 6 are same-side interior angles.



Key Vocabulary

alternate exterior angles

(*ángulos alternos externos*)

alternate interior angles

(*ángulos alternos internos*)

corresponding angles

(*ángulos correspondientes para líneas*)

exterior angle (*ángulo externo*)

interior angle (*ángulos internos*)

remote interior angle

(*ángulo interno remoto*)

same-side interior angles

(*ángulos internos del mismo lado*)

similar (*semejantes*)

transversal (*transversal*)

EXAMPLE 2

Are the triangles similar? Explain your answer.

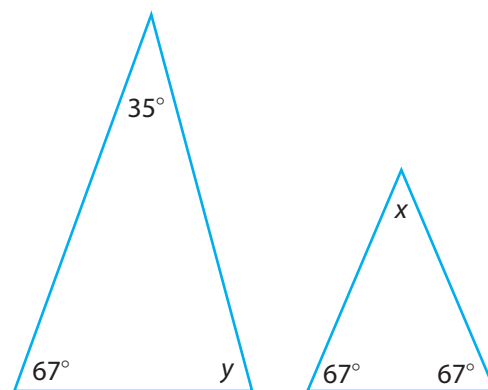
$y = 180^\circ - (67^\circ + 35^\circ)$

$y = 78^\circ$

$x = 180^\circ - (67^\circ + 67^\circ)$

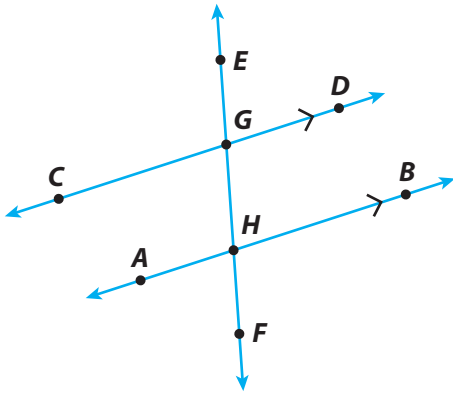
$x = 46^\circ$

The triangles are not similar, because they do not have 2 or more pairs of corresponding congruent angles.



EXERCISES

1. If $m\angle GHA = 106^\circ$, find the measures of the given angles.
(Lesson 11.1)



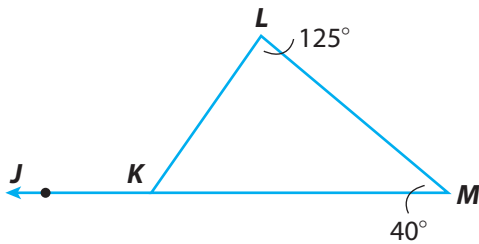
$$m\angle EGC = \underline{\hspace{2cm}}$$

$$m\angle EGD = \underline{\hspace{2cm}}$$

$$m\angle BHF = \underline{\hspace{2cm}}$$

$$m\angle HGD = \underline{\hspace{2cm}}$$

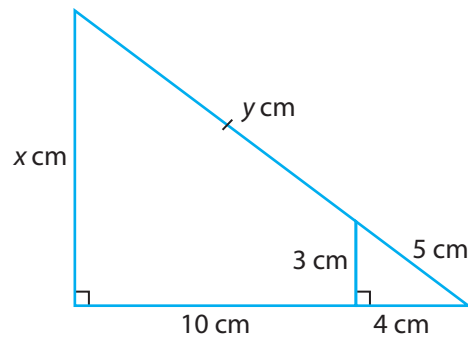
2. Find the measure of the missing angles. (Lesson 11.2)



$$m\angle JKL = \underline{\hspace{2cm}}$$

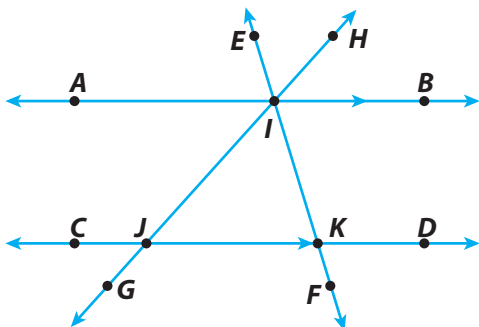
$$m\angle LKM = \underline{\hspace{2cm}}$$

3. Is the larger triangle similar to the smaller triangle?
Explain your answer. (Lesson 11.3)



4. Find the value of x and y in the figure. (Lesson 11.3)

5. If $m\angle CJI = 132^\circ$ and $m\angle EIH = 59^\circ$, find the measures of the given angles. (Lesson 11.1)



$$m\angle IKJ = \underline{\hspace{2cm}}$$

$$m\angle HIB = \underline{\hspace{2cm}}$$

$$m\angle EIJ = \underline{\hspace{2cm}}$$

$$m\angle AIK = \underline{\hspace{2cm}}$$



ESSENTIAL QUESTION

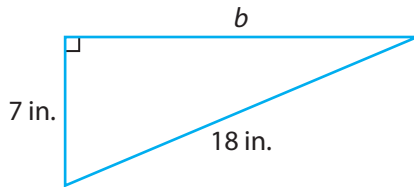
How can you use the Pythagorean Theorem to solve real-world problems?

Key Vocabulary

- hypotenuse (*hipotenusa*)
- legs (*catetos*)
- Pythagorean Theorem (*teorema de Pitágoras*)

EXAMPLE 1

Find the missing side length.
Round your answer to the nearest tenth.



$$a^2 + b^2 = c^2$$

$$7^2 + b^2 = 18^2$$

$$49 + b^2 = 324$$

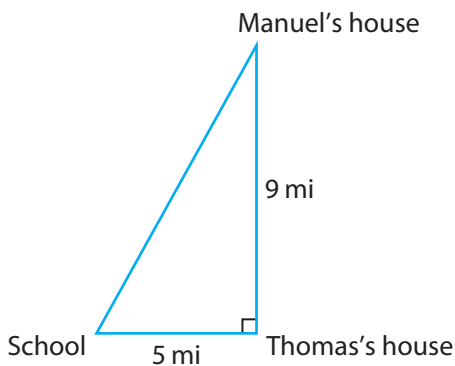
$$b^2 = 275$$

$$b = \sqrt{275} \approx 16.6$$

The length of the leg is about 16.6 inches.

EXAMPLE 2

Thomas drew a diagram to represent the location of his house, the school, and his friend Manuel's house. What is the distance from the school to Manuel's house? Round your answer to the nearest tenth.



$$a^2 + b^2 = c^2$$

$$5^2 + 9^2 = c^2$$

$$25 + 81 = c^2$$

$$c^2 = 106$$

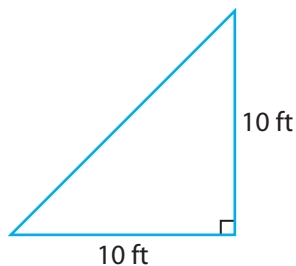
$$c = \sqrt{106} \approx 10.3$$

The distance from the school to Manuel's house is about 10.3 miles.

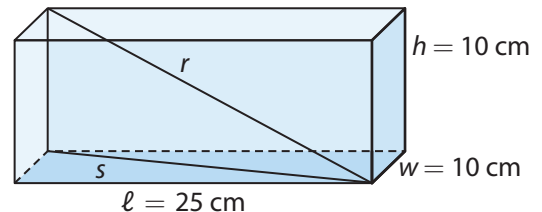
EXERCISES

Find the missing side lengths. Round your answers to the nearest hundredth. (Lesson 12.1)

1.



2.



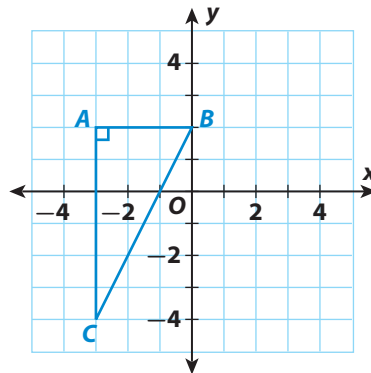
3. Hye Sun has a modern coffee table whose top is a triangle with the following side lengths: 8 feet, 3 feet, and 5 feet. Is Hye Sun's coffee table top a right triangle? (Lesson 12.2)

4. Find the length of each side of triangle ABC. If necessary, round your answers to the nearest hundredth. (Lesson 12.3)

\overline{AB} _____

\overline{BC} _____

\overline{AC} _____



MODULE 13 Volume

ESSENTIAL QUESTION

How can you solve real-world problems that involve volume?

Key Vocabulary

cone (*cono*)

cylinder (*cilindro*)

sphere (*esfera*)

EXAMPLE 1

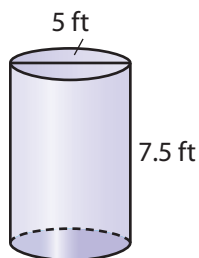
Find the volume of the cistern. Round your answer to the nearest hundredth.

$$V = \pi r^2 h$$

$$\approx 3.14 \cdot 2.5^2 \cdot 7.5$$

$$\approx 3.14 \cdot 6.25 \cdot 7.5$$

$$\approx 147.19$$



The cistern has a volume of approximately 147.19 cubic feet.

EXAMPLE 2

Find the volume of a sphere with a radius of 3.7 cm. Write your answer in terms of π and to the nearest hundredth.

$$V = \frac{4}{3}\pi r^3$$

$$\approx \frac{4}{3} \cdot \pi \cdot 3.7^3$$

$$\approx \frac{4}{3} \cdot \pi \cdot 50.653$$

$$\approx 67.54\pi$$

$$V = \frac{4}{3}\pi r^3$$

$$\approx \frac{4}{3} \cdot 3.14 \cdot 3.7^3$$

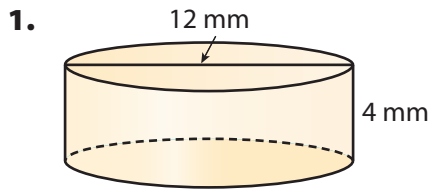
$$\approx \frac{4}{3} \cdot 3.14 \cdot 50.653$$

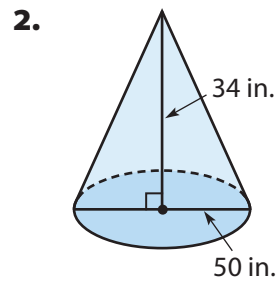
$$\approx 212.07$$

The volume of the sphere is approximately $67.54\pi \text{ cm}^3$, or 212.07 cm^3 .

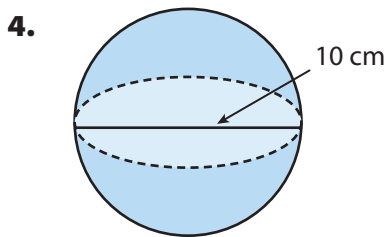
EXERCISES

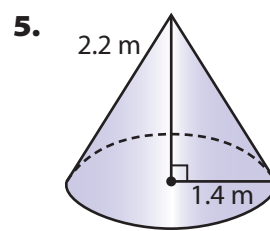
Find the volume of each figure. Round your answers to the nearest hundredth. (Lessons 13.1, 13.2, 13.3)



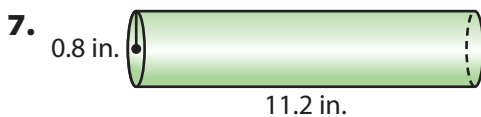


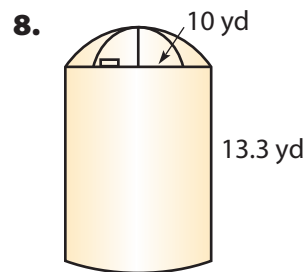
3. Find the volume of a ball with a radius of 1.68 inches. _____





6. A round above-ground swimming pool has a diameter of 15 ft and a height of 4.5 ft. What is the volume of the swimming pool? _____





9. A paper cup in the shape of a cone has a height of 4.7 inches and a diameter of 3.6 inches. What is the volume of the paper cup? _____

Unit 5 Performance Tasks

1. **CAREERS IN MATH** **Hydrologist** A hydrologist needs to estimate the mass of water in an underground aquifer, which is roughly cylindrical in shape. The diameter of the aquifer is 65 meters, and its depth is 8 meters. One cubic meter of water has a mass of about 1000 kilograms.

- a. The aquifer is completely filled with water. What is the total mass of the water in the aquifer? Explain how you found your answer. Use 3.14 for π and round your answer to the nearest kilogram.

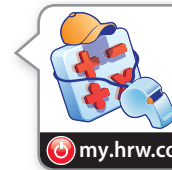
- b. Another cylindrical aquifer has a diameter of 70 meters and a depth of 9 meters. The mass of the water in it is 27×10^7 kilograms. Is the aquifer totally filled with water? Explain your reasoning.

2. From his home, Myles walked his dog north 5 blocks, east 2 blocks, and then stopped at a drinking fountain. He then walked north 3 more blocks and east 4 more blocks. It started to rain so he cut through a field and walked straight home.

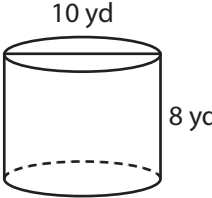
- a. Draw a diagram of his path.

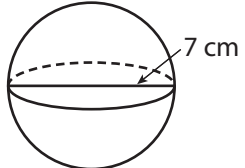


- b. How many blocks did Myles walk in all? How much longer was his walk before it started to rain than his walk home?



Selected Response

- Which of the following angle pairs formed by a transversal that intersects two parallel lines are not congruent?
 - alternate interior angles
 - adjacent angles
 - corresponding angles
 - alternate exterior angles
- The measures of the three angles of a triangle are given by $3x + 1$, $2x - 3$, and $9x$. What is the measure of the smallest angle?
 - 13°
 - 23°
 - 29°
 - 40°
- Using 3.14 for π , what is the volume of the cylinder?
 
 - 200 cubic yards
 - 628 cubic yards
 - 1256 cubic yards
 - 2512 cubic yards
- Which of the following is **not** true?
 - $\sqrt{36} + 2 > \sqrt{16} + 5$
 - $5\pi < 17$
 - $\sqrt{10} + 1 < \frac{9}{2}$
 - $5 - \sqrt{35} < 0$
- A pole is 65 feet tall. A support wire is attached to the top of the pole and secured to the ground 33 feet from the base of the pole. Find the approximate length of the wire.
 - 32 feet
 - 73 feet
 - 56 feet
 - 60 feet

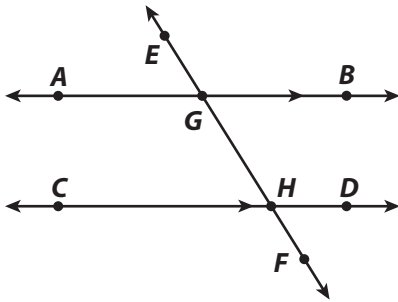
- Using 3.14 for π , what is the volume of the sphere to the nearest tenth?
 
 - 205.1 cm^3
 - 1077 cm^3
 - 179.5 cm^3
 - 4308.1 cm^3

- Which set of lengths are **not** the side lengths of a right triangle?
 - 28, 45, 53
 - 13, 84, 85
 - 36, 77, 85
 - 16, 61, 65
- Which statement describes the solution of a system of linear equations for two lines with different slopes and different y -intercepts?
 - one nonzero solution
 - infinitely many solutions
 - no solution
 - solution of 0
- What is the side length of a cube that has a volume of 729 cubic inches?
 - 7 inches
 - 8 inches
 - 9 inches
 - 10 inches
- What is the solution to the system of equations?

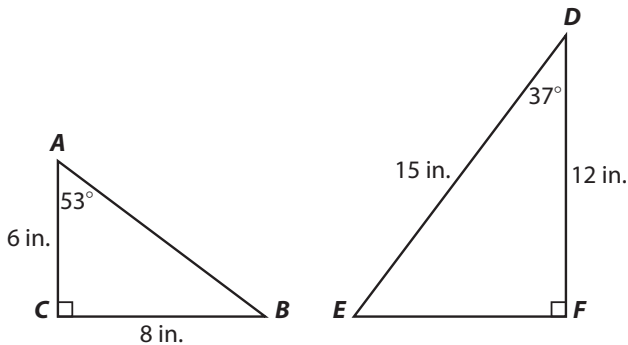
$$\begin{cases} x + 3y = 5 \\ 2x - y = -4 \end{cases}$$
 - no solution
 - infinitely many solutions
 - (2, 1)
 - (-1, 2)

Mini-Tasks

11. In the figure shown, $m\angle AGE = (5x - 7)^\circ$ and $m\angle BGH = (3x + 19)^\circ$.



- Find the value of x . _____
 - Find $m\angle AGE$. _____
 - Find $m\angle GHD$. _____
12. Tom drew two right triangles as shown with angle measures to the nearest whole unit.



- Find the length of \overline{AB} in triangle ABC .

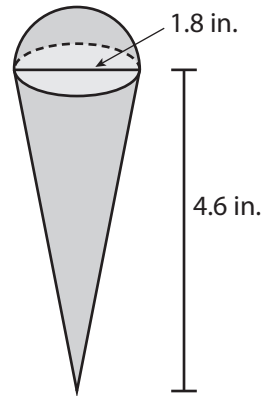
- Find the length of \overline{EF} in triangle DEF .

- Are the triangles similar? Explain your answer.



Read graphs and diagrams carefully. Look at the labels for important information.

13. In the diagram, the figure at the top of the cone is a hemisphere.



- What is the volume of the cone? Round your answer to the nearest hundredth.

- What is the volume of the hemisphere on the top of the cone? Round your answer to the nearest hundredth.

- What would be the radius of a sphere with the same total volume as the figure? Explain how you found your answer.

